Linear Programming (Step by Step - **Full Process**)

- Inequalities ($\geq$, $\leq$)
- Objective Function ($z = ax+by$)
- Diagram
  - Feasible
  - No Feasible Region ✗
- Corner Points
- Table (M/m)
  - Bounded (M/m) ✓
  - Unbounded (M/m)
    - $ax+by > M$
      - No Common ✓
      - Common ✗
    - $ax+by < m$ (Diagram)
      - No Common ✓
      - Common ✗

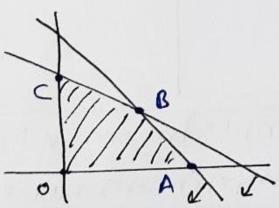
IOD.FCT.22

(Table)

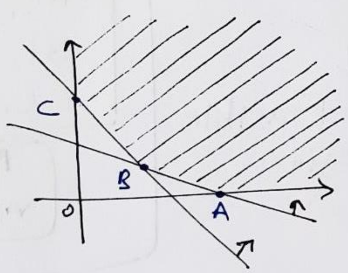
Corner Points	$Z = ax+by$
A(,)	—
B(,)	—
C(,)	—

M = maximum Value
m = minimum value

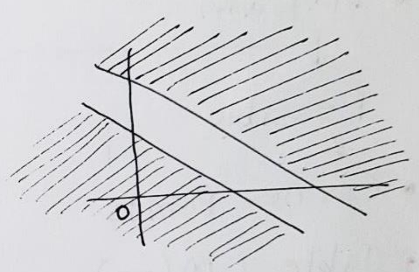
Objective Function $z = ax+by$ (जिसका minimum या maximum पता करना है)



Feasible
(Bounded)



Feasible
(Unbounded)



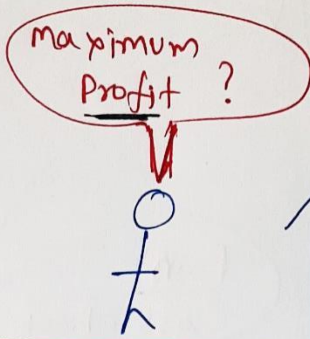
No Feasible Region

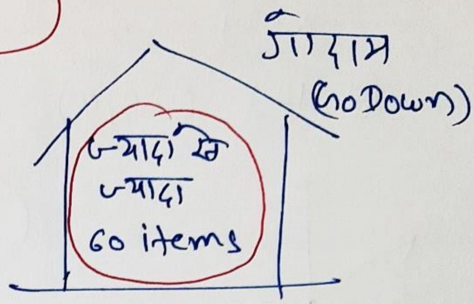
(रैखिक प्रोग्रामन)

Practical Situation.

(Shop keeper)

(Tables + Chairs)





$\boxed{₹\ 50000}$

Cost of 1 table = ₹ 2500	Profit on 1 table = ₹ 250
Cost of 1 chair = ₹ 500	Profit on 1 chair = ₹ 75

Let No. of tables = x
No. of Chairs = y
→ (Decision Variables)

Total Cost = $2500x + 500y \le 50000$
Space = $x + y \le 60$
→ Constraints ✓ (अवरोध) (पाबंदी)

$\boxed{\text{Profit } (z) = 250x + 75y}$ → maximum

Objective function → M (maximum) → m (minimum)

0 table, y chairs → $0 + \cancel{500}y \le \cancel{50000}\ 100$

$y \le 100$

$x = 0$
$y = 60$

Profit (z)
$= 250 \times 0 + 75 \times 60$
$= 4500$

x tables, 0 chair → $\cancel{2500}x + 0 \le \cancel{50000}\ 20$

$x \le 20$

$x + 0 \le 60$

$x = 20$
$y = 0$

Profit (z)
$= 250x + 75y$
$= 250 \times 20 + 0$
$= 5000$

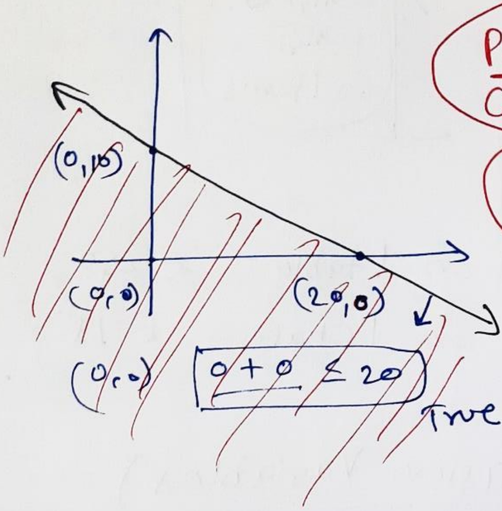
How to solve linear Inequalities (in 2 variables (x) (y))?

$\boxed{x+2y \le 20}$

$x+2y=20$

$(0,10)$ $(20,0)$

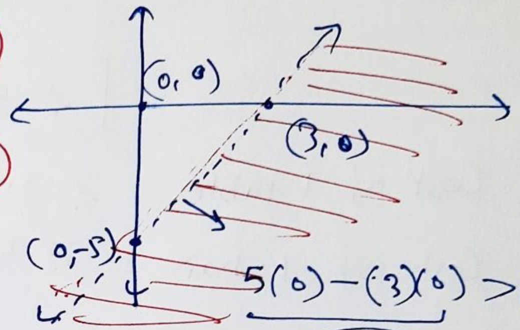
Point
Cross check
$(0,0)$
Generally



$\boxed{5x-3y \ge 15}$

$5x-3y=15$

$(0,-5)$, $(3,0)$

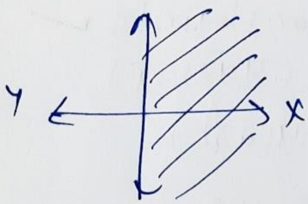


$5(0)-(3)(0) > 15$

$0 > 15$

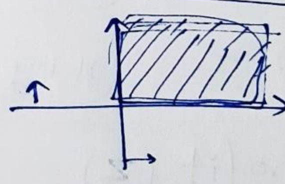
False

e.g. $x \ge 0$



$y \ge 0$

$\boxed{x \ge 0 \ \& \ y \ge 0}$ Common



How to Solve Questions of Linear programming ?

Full Process - Step by Step

Corner Points	$Z = ax + by$
A (,) (x, y)	
B (,)	
C (,)	

Table

M = maximum value

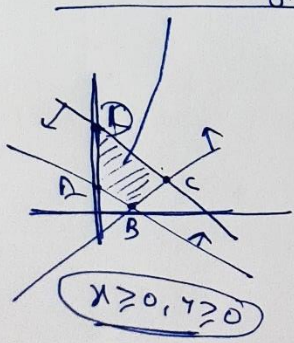
m = minimum value

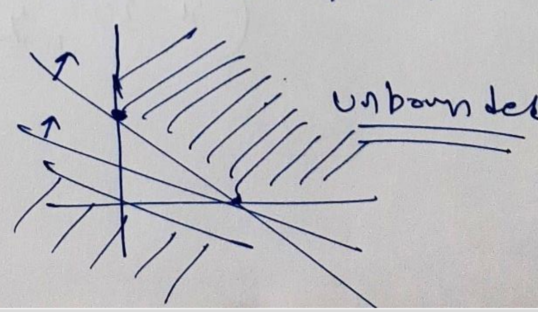
- Inequalities ($\geq, \leq$)
- Objective Function ($Z = ax + by$)
- Diagram
  - Feasible
  - No Feasible Region ✗
- Corner points (x, y)
- Table
  - Bounded (M/m) ✓
  - Unbounded (M/m)
    - $ax + by > M$
      - No Common ✓
      - Common ✗
    - $ax + by < m$
      - No Common ✓
      - Common ✗

Remember $\Rightarrow$ $\boxed{\text{IOD. FCT. 22}}$

Objective function → वो quantity जिसका maximum | minimum पता करना है।

Feasible Region → सब का common (Graph है)





e.g. Solve the following linear programming problem graphically: $\boxed{\text{Maximise } z = 4x+y}$ Objective Function.

Subject to the constraints: $x+y \le 50$ — ①

$3x+y \le 90$ — ②

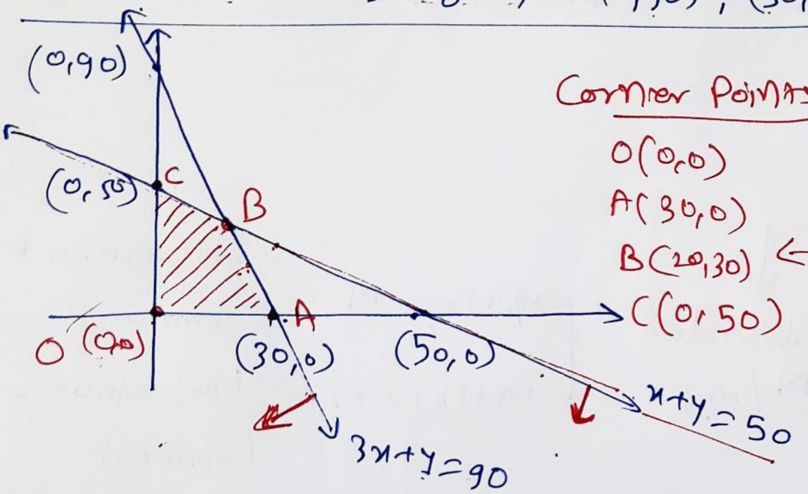
$x \ge 0, y \ge 0$ — ③

I-quadrant

Ans. Diagram.

$\boxed{x+y \le 50 \longrightarrow x+y=50 \quad (0,50),\ (50,0)}$

$\boxed{3x+y \le 90 \longrightarrow 3x+y=90 \quad (0,90),\ (30,0)}$



Corner Points

O(0,0)

A(30,0)

B(20,30)

C(0,50)

For Point (B)

$3x+\cancel{y}=90$

$x+\cancel{y}=50$

$- \quad -$

$2x = 40$

$x=20$

$y=30$

Table

Corner Points	$z=4x+y$
O (0,0)	0
A (30,0)	120
B (20,30)	80+30=110
C (0,50)	50

M=120 maximum

$\therefore$ maximum value of $(z=4x+y) = 120$

$(x=30,\ y=0)$

e.g. A Furniture dealer deals in only two items - tables & chairs. He has ₹ 50000 to invest and has storage space of at most 60 pieces. A table costs ₹ 2500 and a chair ₹ 500. He estimates that from the sale of one table, he can make a profit of ₹ 250 and that from the sale of one chair a profit of ₹ 75. How many tables and chairs should be bought so as to maximise his profit? Also find maximum profit.

Let no. of tables $= x \geq 0$
no. of chairs $= y \geq 0$

No.	Cost	Profit
x	2500	250
y	500	75

Constraints (Inequalities)

$2500x + 500y \leq 50000$ (Investment Constraint)

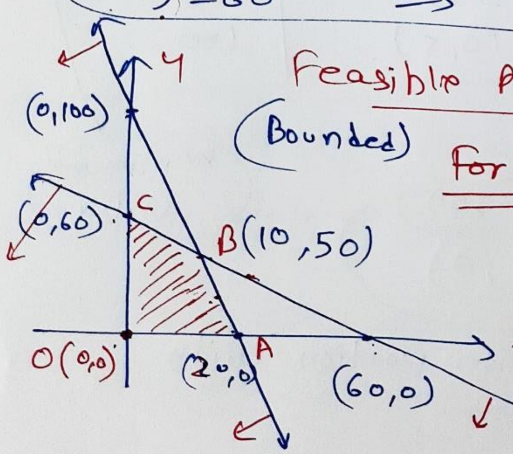
$x + y \leq 60$ (Space Constraint)

Objective fn:
$$\boxed{\text{Profit }(Z) = 250x + 75y}$$

Diagram:

$2500x + 500y \leq 50000 \Rightarrow 2500x + 500y = 50000$ (0,100) (20,0)

$x + y \leq 60 \Rightarrow x + y = 60$ (0,60), (60,0)



Feasible Region OABC (Bounded)

For 'B'

$x + y = 60$
$- \; 5x + y = 100$
$-4x = -40$
$x = 10$

$2500x + 500y = 50000$
$\Rightarrow (5x + y = 100)$

Table Corner Points	Profit $Z = 250x + 75y$
O (0,0)	0
A (20,0)	5000
B (10,50)	6250 → M
C (0,60)	4500

No. of table $= x = 10$
No. of chairs $= y = 50$

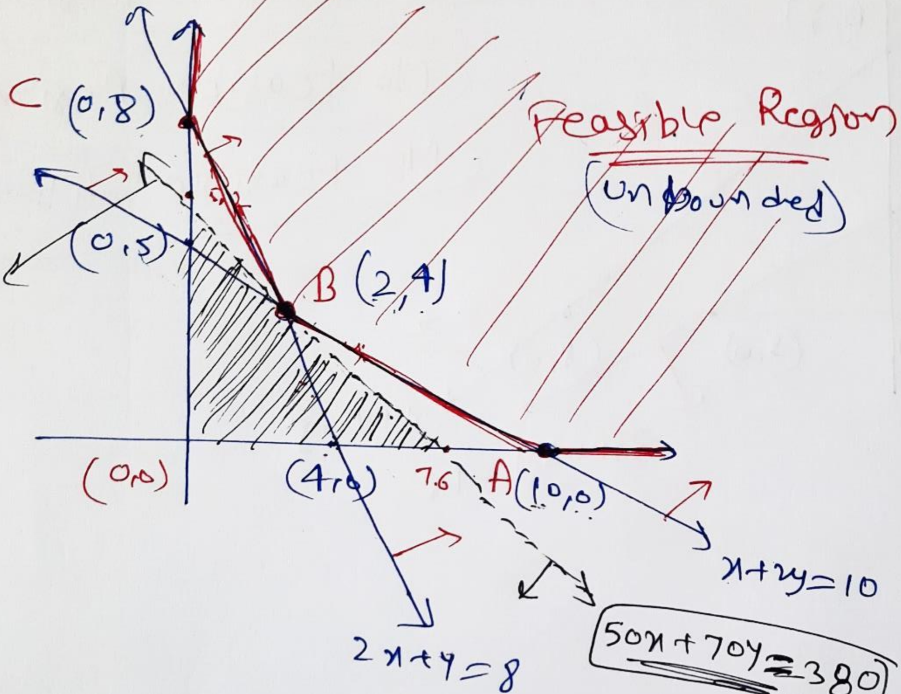
maximum Profit

e.g. Determine graphically the minimum value of the objective function $\boxed{Z = 50x + 70y}$ subject to constraints :

$2x + y \geq 8 \rightarrow 2x + y = 8 \quad (0,8), (4,0)$

$x + 2y \geq 10 \rightarrow x + 2y = 10 \quad (0,5), (10,0)$

$\boxed{x \geq 0, y \geq 0} \rightarrow$ I.



For (B)

$x + 2y = 10$

$2x + y = 8$

$2x + 4y = 20$

$-3y = -12$

$y = 4$

$x = 2$

Table

Corner Point	$Z = 50x + 70y$
A (10,0)	500
B (2,4)	380 ← m = minimum value.
C (0,8)	560

∵ Our feasible region is unbounded.

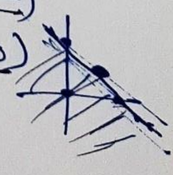
New Inequality, $\boxed{50x + 70y < 380}$ → Diagram.

Dotted line

$50x + 70y = 380$

(0, 5.2)

(7.6, 0)



(2,4)

$100 + 280 = 380$

$380 = 380$

∴ $50x + 70y < 380$ & Feasible Region has no common portion.

∴ minimum value of $Z = 380$

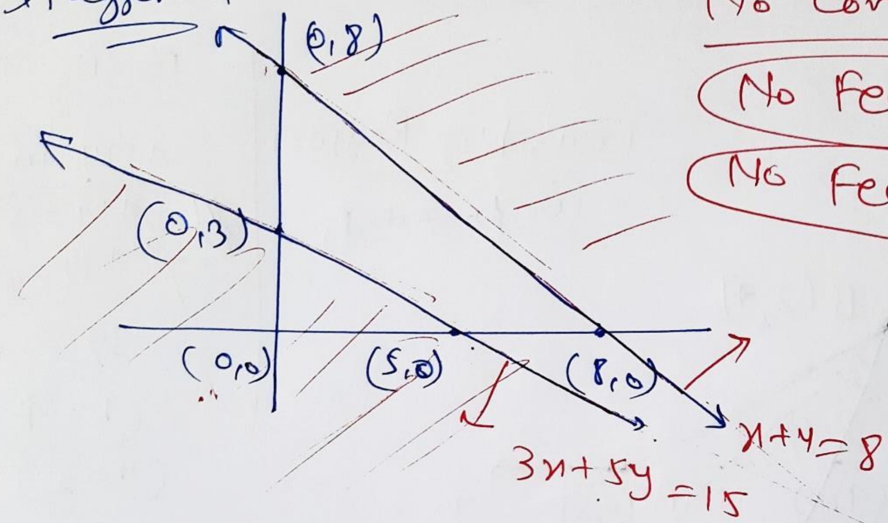
e.g. Minimize $\boxed{Z = 3x+2y}$ subject to constraints

$x+y \geq 8$ → $x+y=8 \rightarrow (0,8), (8,0)$

$3x+5y \leq 15$ → $3x+5y=15$ $(0,3), (5,0)$

$x \geq 0, y \geq 0$ → I Quad

Diagram



(No Common Region

No Feasible Region

No Feasible Solution

Exercise 12.1

Solve the following Linear Programming Problems graphically:

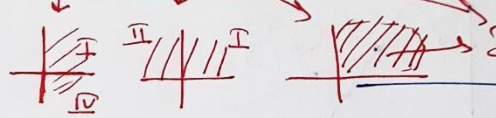
Q.1 Maximise $\boxed{Z = 3x + 4y}$

Subject to the constraints : $x + y \le 4$, $x \ge 0$, $y \ge 0$.

Ans:

Inequalities. $\boxed{x + y \le 4} \Rightarrow x + y = 4$ $(0,4)$, $(4,0)$

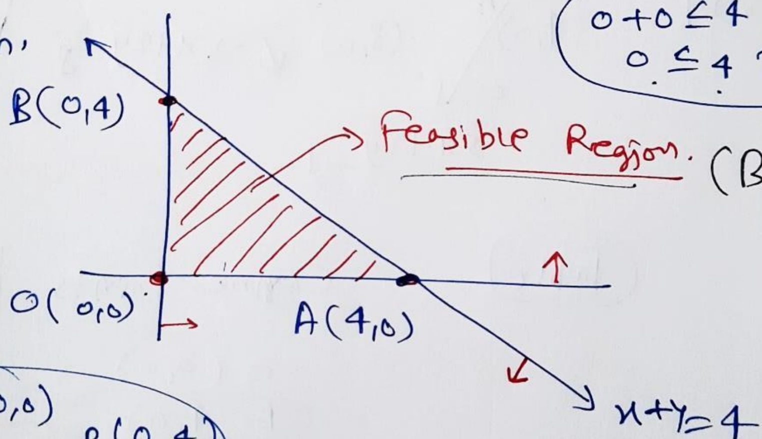
$\boxed{x \ge 0, y \ge 0}$



Objective Function

$(Z = 3x + 4y)$

Diagram.



$0 + 0 \le 4$

$0 \le 4$ True

Corner Points. O(0,0) A(4,0) B(0,4)

Table. (M/m)

(maximize)

Corner Points	$Z = 3x + 4y$
O (0,0)	0
A (4,0)	12
$\boxed{B\ (0,4)}$	$\boxed{16}$ ← M = maximum

∵ Since Feasible region is bounded,

∴ maximum value of $(Z = 3x + 4y) = 16$

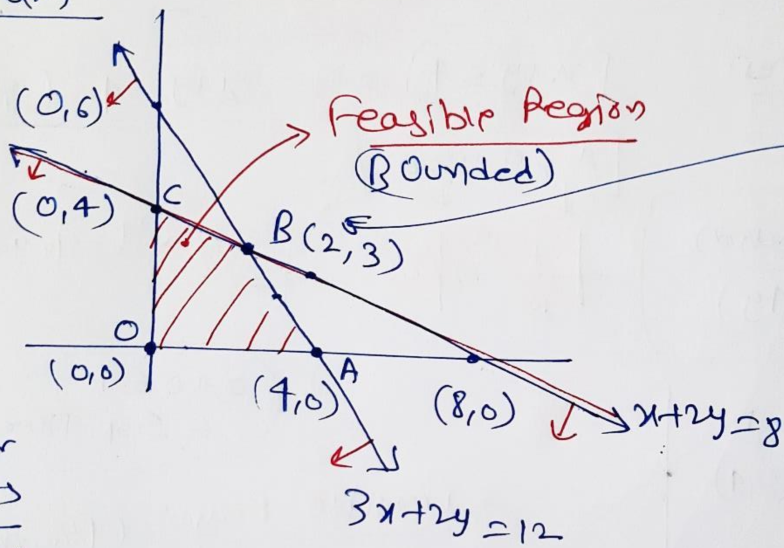
at $(0,4)$

Q.2 Minimize $\boxed{Z = -3x + 4y}$ ← objective fn.

Subject to constraints:

Inequalities
$$\begin{cases} x + 2y \le 8 \rightsquigarrow x + 2y = 8 \quad (0,4), (8,0) \\ 3x + 2y \le 12 \rightsquigarrow 3x + 2y = 12 \quad (0,6), (4,0) \\ x \ge 0, y \ge 0. \longrightarrow \text{I quadrant} \end{cases}$$

Diagram



For 'B'

$$3x + \cancel{2y} = 12$$
$$-\; x + \cancel{2y} = 8$$
$$2x = 4$$
$$x = 2$$
$$y = 3$$

Corner Points

O (0,0)
A (4,0)
C (0,4)
B (2,3)

Table

Corner Points	minimize $Z = -3x + 4y$
O (0,0)	0
A (4,0)	−12 = m
B (2,3)	6
C (0,4)	16

minimum value

minimum value of ($Z = -3x + 4y$)

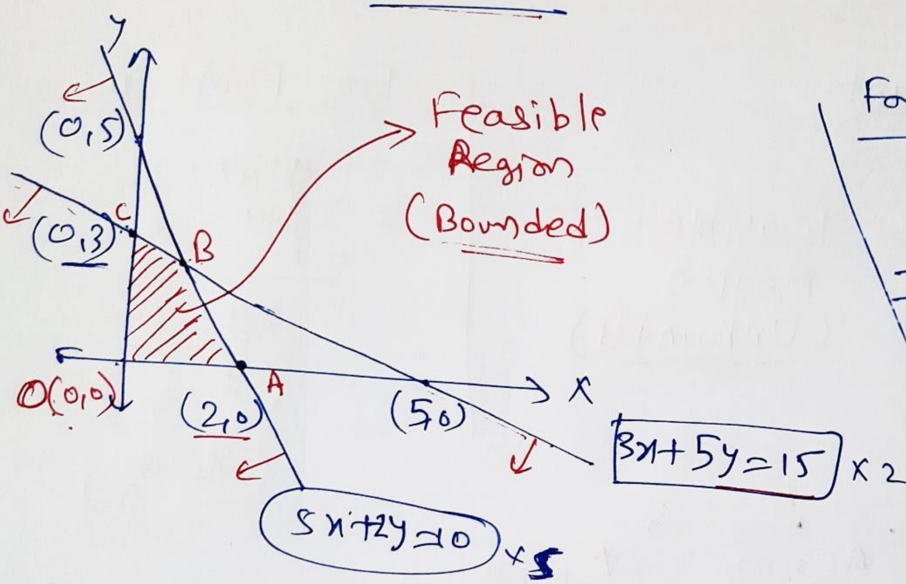
$= -12$ at $(4,0)$

Q.3 maximize $\boxed{Z = 5x+3y}$ subject to objective function

$3x+5y \le 15, \longrightarrow 3x+5y=15 \quad (0,3), (5,0)$

$5x+2y \le 10, \longrightarrow 5x+2y=10 \quad (0,5), (2,0)$

$\boxed{x \ge 0,\ y \ge 0} \rightarrow$ I-quadrant



For Point B

$$\begin{array}{r} 6x+\cancel{10y}=30 \\ 25x+\cancel{10y}=50 \\ \hline -19x=-20 \end{array}$$

$$x = \frac{20}{19}$$

$$y = \frac{45}{19}$$

B

Table

Corner Points	$Z = 5x+3y$
O (0,0)	0
A (2,0)	10
B $\left(\frac{20}{19}, \frac{45}{19}\right)$	$235/19$ = M
C (0,3)	9

(maximum value)

- Since feasible region is bounded

$\therefore$ maximum value of $(Z = 5x+3y) = \frac{235}{19}$

at $\left(\frac{20}{19}, \frac{45}{19}\right)$

Q.4 Minimise $\boxed{Z = 3x + 5y}$ such that

→ objective Function

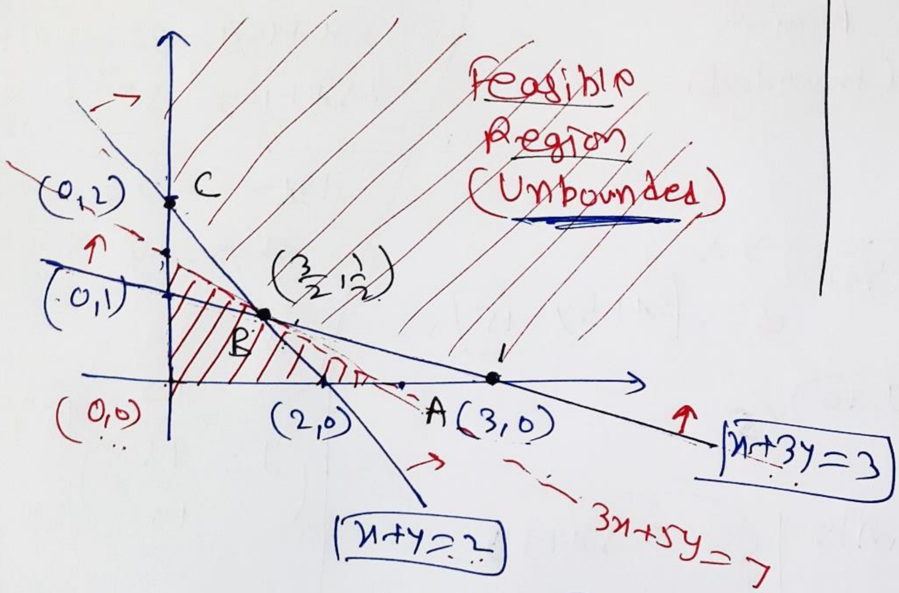
Ineq.
$x + 3y \geq 3 \longrightarrow x + 3y = 3 \quad (0,1), (3,0)$

$x + y \geq 2 \longrightarrow x + y = 2 \quad (0,2), (2,0)$

$x, y \geq 0$

↓

I-quadrant.



For Point (B)

$$\begin{aligned} x + 3y &= 3 \\ x + y &= 2 \\ \hline 2y &= 1 \\ y &= \tfrac{1}{2} \\ x &= \tfrac{3}{2} \end{aligned}$$

Table.

Corner Points.	$Z = (3x + 5y)$ (Minimise)
A (3,0)	9
B $(\frac{3}{2}, \frac{1}{2})$	7 ← m = minimum value.
C (0,2)	10

Since feasible region is unbounded.

$\Rightarrow \boxed{3x + 5y < 7}$ Diagram $3x + 5y = 7 \quad (0, 1.4)$, $(2.3, 0)$

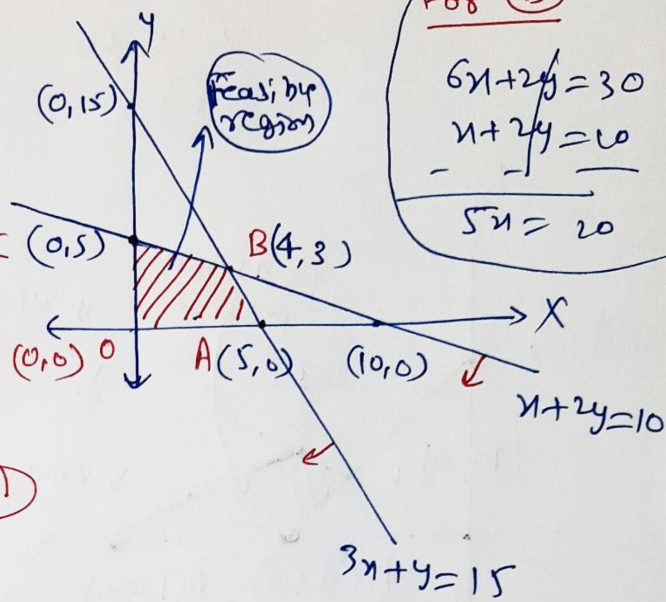
$(\frac{3}{2}, \frac{1}{2})$ will lie on

$\Rightarrow \because$ There is no Common Part of $3x + 5y < 7$ and Feasible region $\longrightarrow \therefore$ min. $(Z) = 7$ at $(\frac{3}{2}, \frac{1}{2})$

Q.5 Maximize $Z = 3x + 2y$ subject to

$x + 2y \le 10$, $3x + y \le 15$, $x, y \ge 0$.

$x + 2y = 10$	$3x + y = 15$
$(0,5)$, $(10,0)$	$(0,15)$, $(5,0)$



For (B)

$$6x + 2y = 30$$
$$x + 2y = 10$$
$$5x = 20$$

Table — maximise

Corner Points	$Z = 3x + 2y$
O $(0,0)$	0
A $(5,0)$	15
B $(4,3)$	18 = M
C $(0,5)$	10

maximum value of $Z = 18$ at $(4,3)$ (Bounded)

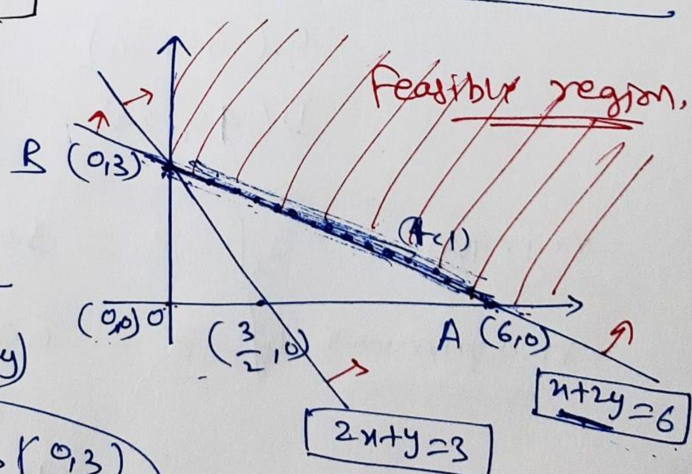
Q.6 Minimize $Z = x + 2y$ subject to $2x + y \ge 3$, $x + 2y \ge 6$, $x \ge 0, y \ge 0$.

Show that the minimum of Z occurs at more than two points.

$2x + y = 3$	$x + 2y = 6$
$(0,3)$, $(\frac{3}{2},0)$	$(0,3)$ $(6,0)$

Table

Corner Points	$Z = x + 2y$
A $(6,0)$	6
B $(0,3)$	6



minimum value of $(Z = x + 2y) = 6$ at A$(6,0)$ & B$(0,3)$

at all points lying on line segment $\overline{AB}$

Q. 7 Minimise and maximise $Z = 5x + 10y$

Subject to $x + 2y \le 120 \rightsquigarrow x + 2y = 120 \quad (0, 60), (120, 0)$

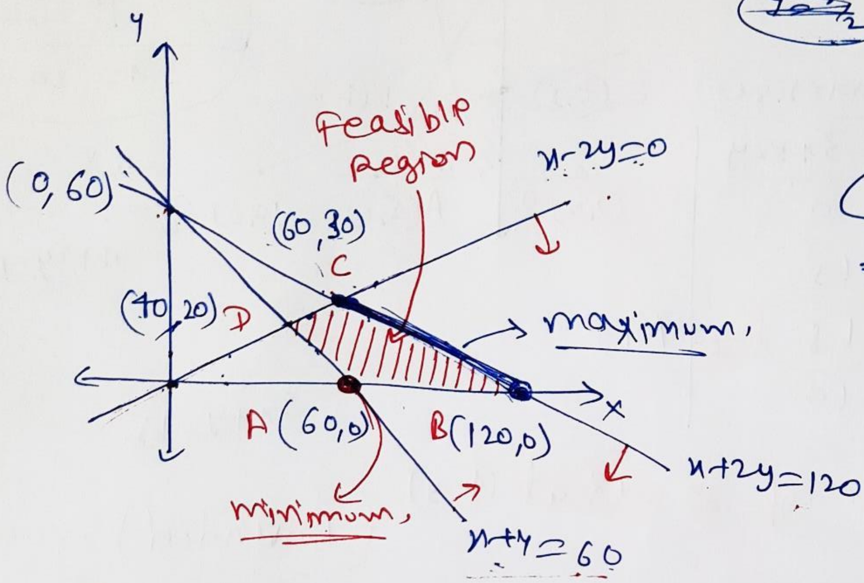
$x + y \ge 60 \rightsquigarrow x + y = 60 \quad (0, 60), (60, 0)$

$x - 2y \ge 0 \rightsquigarrow x - 2y = 0 \quad (0, 0), (1, \frac{1}{2})$

$\boxed{x, y \ge 0}$

$x = 2y$

$y = \frac{x}{2}$



For C

$x = 2y$

$x + 2y = 120$

$\Rightarrow 4y = 120$

$y = 30$

For D

$x = 2y$

$x + y = 60$

$3y = 60$

$y = 20$

Table.

Corner Points	$Z = 5x + 10y$
A (60, 0)	300 ← m = minimum
B (120, 0)	600 } → M = maximum
C (60, 30)	600
D (40, 20)	400

minimum of $Z = 300$ at (60, 0)

maximum of $Z = 600$ at all points lying on the line segment joining (120, 0) & (60, 30).

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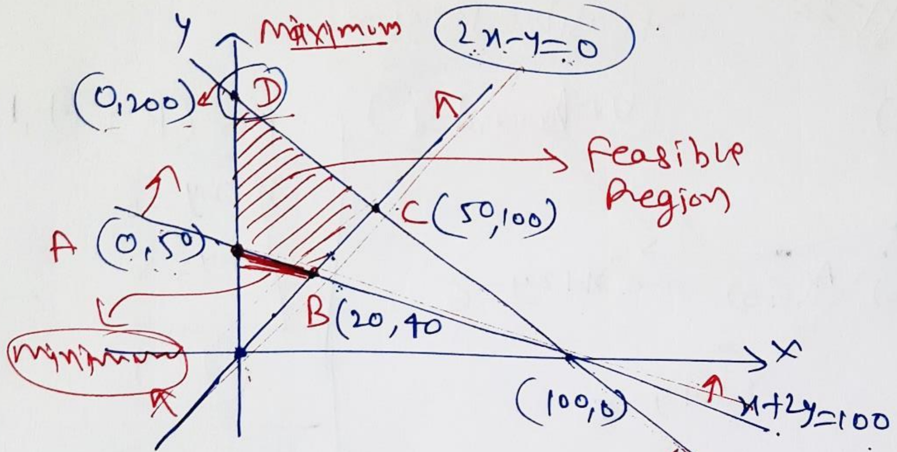
Q.8 Minimise and maximise $\boxed{Z = x + 2y}$ subject to

$x + 2y \geq 100$, $2x - y \leq 0$, $2x + y \leq 200$, $x, y \geq 0$.

$x + 2y = 100$	$2x - y = 0$	$2x + y = 200$
$(0, 50), (100, 0)$	$(0,0), (1,2)$	$(0, 200), (100, 0)$



For (B)

$y = 2x$

$x + 2y = 100$

$5x = 100$

$x = 20$

For (C)

$y = 2x$

$2x + y = 200$

$4x = 200$

$x = 50$

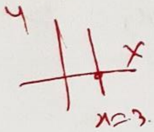
Table

Corner Points	$Z = x + 2y$	
A (0,50)	100	m = minimum.
B (20,40)	100	
C (50,100)	250	
D (0,200)	400	M = maximum.

Minimum of $Z = 100$ at all points lying on line segment joining (0, 50), (20, 40)

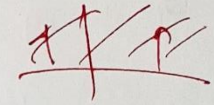
maximum of $Z = 400$ at (0, 200).

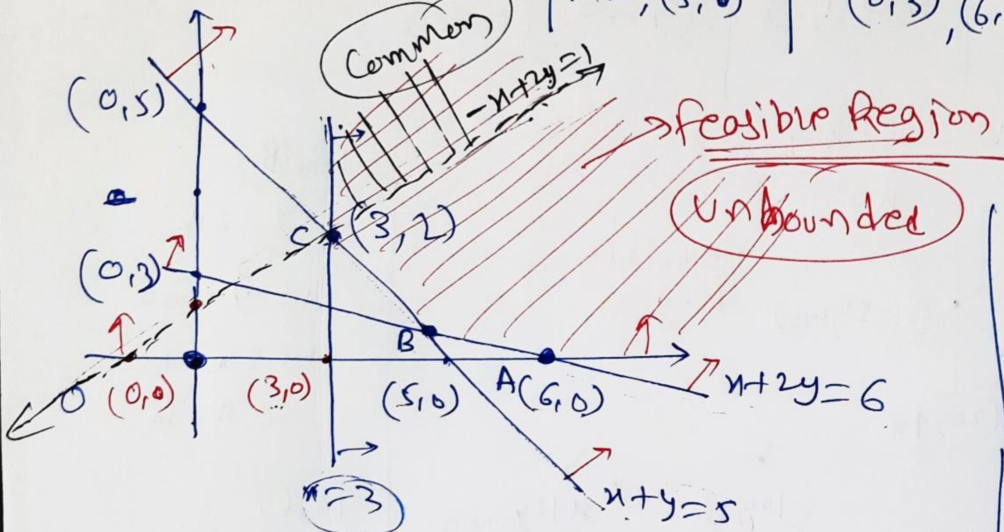
Q.9 Maximise $Z = -x + 2y$, subject to the constraints: $x \geq 3$, $x + y \geq 5$, $x + 2y \geq 6$, $y \geq 0$.



$x = 3$

$x + y = 5$	$x + 2y = 6$
$(0,5), (5,0)$	$(0,3), (6,0)$





For (B) (4,1)

$x + 2y = 6$

$x + y = 5$

$\boxed{y = 1}$

For (C)

Table: ~~Corner~~

Corner Points	$Z = -x + 2y$ (maximise)
A (6, 0)	-6
B (4, 1)	-2
C (3, 2)	1 ← M = Maximum,

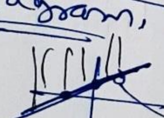
But feasible ↓ unbounded

Let $(-x + 2y) > 1$

$0 + 0 > 1$

$0 > 1$

Diagram,



$-x + 2y = 1$, $(0, \frac{1}{2})$, $(-1, 0)$

(3,2)

Since $(-x + 2y > 1)$ & feasible region has some common portion.

∴ no maximum value of Z.

Q.10 Maximise $Z = x+y$, subject to

$\underbrace{x-y \le -1}$, $\underbrace{-x+y \le 0}$, $\underbrace{x, y \ge 0}$.

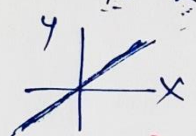
I-quads

$x-y=-1$

$(0,1), (-1,0)$

$-x+y=0$

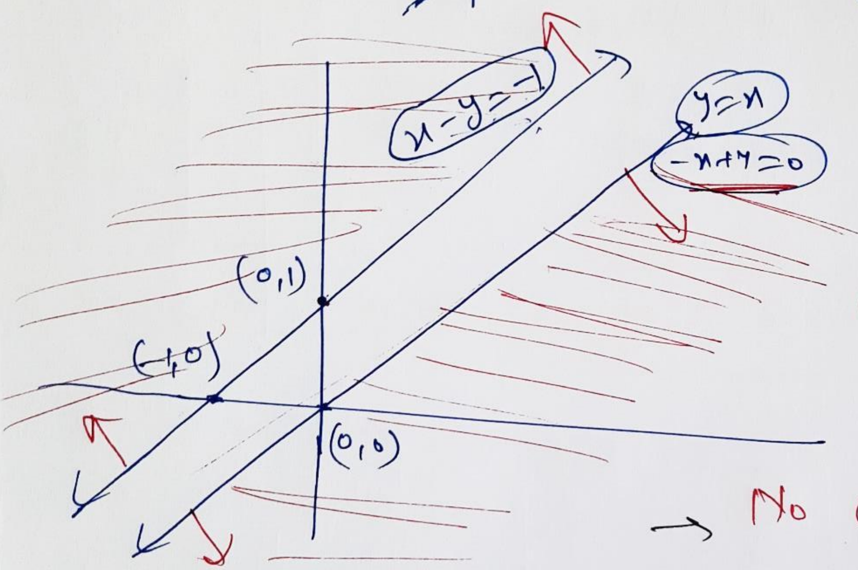
$y=x$



$y = mx + c$

$y = x+1 \to m_1 = 1$

$y = x \to m_2 = 1$



$0-0 \le -1$

$0 \le -1$ ✗

$-x+y \le 0$

$0+1 \le 0$ ✗

→ No Common Region

→ No Feasible Region

→ No Feasible Solution

→ No maximum

Exercise 12.2

Q.1 let quantity of food 'P' = x kg

'Q' = y kg

	Cost (₹/kg)	A (units/kg)	B (units/kg)
P	60	3	5
Q	80	4	2

Vitamin 'A' ≥ 8 units.

Vitamin 'B' ≥ 11 units.

Inequality

$3x + 4y \geq 8$ (Vit 'A' Constraint)

$5x + 2y \geq 11$ (Vit 'B' Constraint)

$x, y \geq 0$

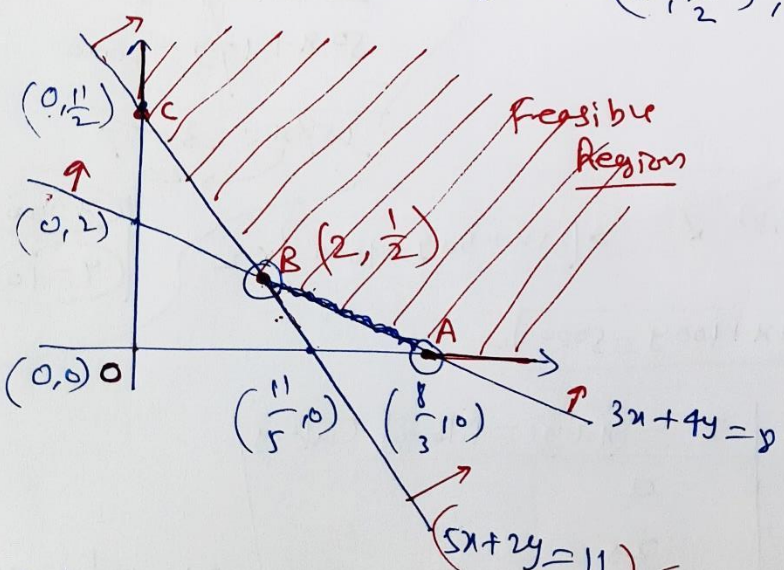
Objective Function $\boxed{\text{Cost } (Z) = 60x + 80y}$ (minimise)

$3x + 4y \geq 8 \longrightarrow 3x + 4y = 8 \quad (0, 2), \left(\frac{8}{3}, 0\right)$

$5x + 2y \geq 11 \longrightarrow 5x + 2y = 11 \quad \left(0, \frac{11}{2}\right), \left(\frac{11}{5}, 0\right)$

$x, y \geq 0$

I-quadrant



For (B)

$$3x + 4y = 8$$
$$10x + 4y = 22$$
$$-7x = -14$$

$x = 2$, $y = \frac{1}{2}$

Table,

Corner Points	$Z = 60x + 80y$
A $\left(\frac{8}{3}, 0\right)$	160
B $\left(2, \frac{1}{2}\right)$	160
C $\left(0, \frac{11}{2}\right)$	440

160, 160 } minimum value of cost

Minimum Cost = 160 at all points lying on line segment joining $\left(\frac{8}{3}, 0\right)$ & $\left(2, \frac{1}{2}\right)$

Q.2 let no. of cakes of type$_I$ = $x \geq 0$

type$_{II}$ = $y \geq 0$

$5\,kg = 5000\,gm$

$(1\,kg = 1000\,gm)$

	Floor	fat
Type-I	200 gm	25 gm
Type-II	100 gm	50 gm

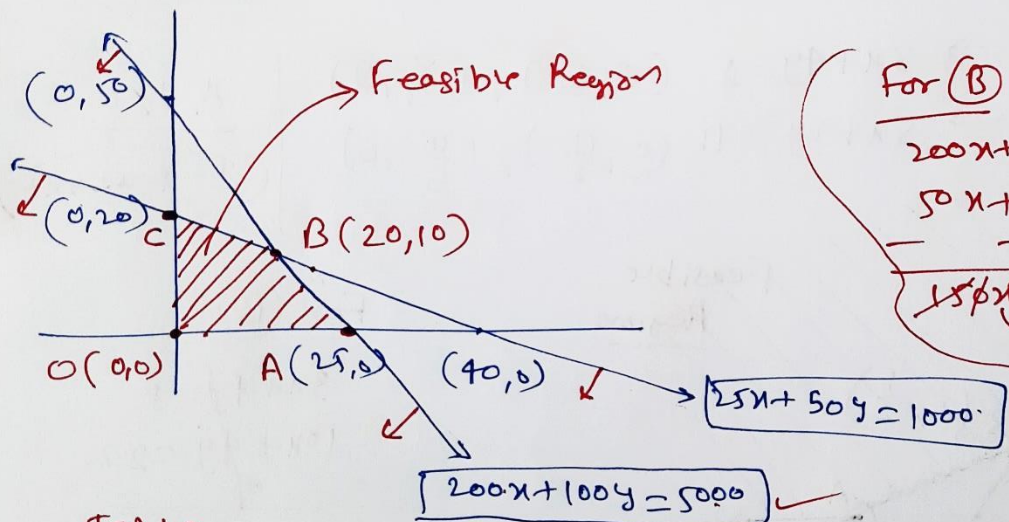
Constrains.

$x \geq 0, y \geq 0 \rightarrow$ I-quad.

(flour) $200x + 100y \leq 5000 \rightarrow 200x + 100y = 5000$
$(0,50), (25,0)$

(Fat) $25x + 50y \leq 1000 \rightarrow 25x + 50y = 1000$
$(0,20), (40,0)$

Objective Function = No. of Cakes $(z) = x + y$ (maximum)



For (B)

$200x + 100y = 5000$

$50x + 100y = 2000$ ($25x+50y=1000$, $\times 2$)

$\overline{150x = 3000}$

$x = 20$

$y = 10$

Table.

Corner Points	$z = x + y$ = No. of Cakes
O (0,0)	0
A (25,0)	25
B (20,10)	30 = Maximum No. of cakes
C (0,20)	20

Type$_I$ = 20 cakes

Type$_{II}$ = 10 Cakes

Q.3

	Quantity	Machine time (hours)	Craftman's time (hours)	Profit
Tennis Racket	x	1.5	3	20 (₹)
Cricket Bat	y	3	1	10 (₹)
	$x+y$	maximum (42 hrs)	max. (24 hrs)	Total Profit $= 20x+10y$

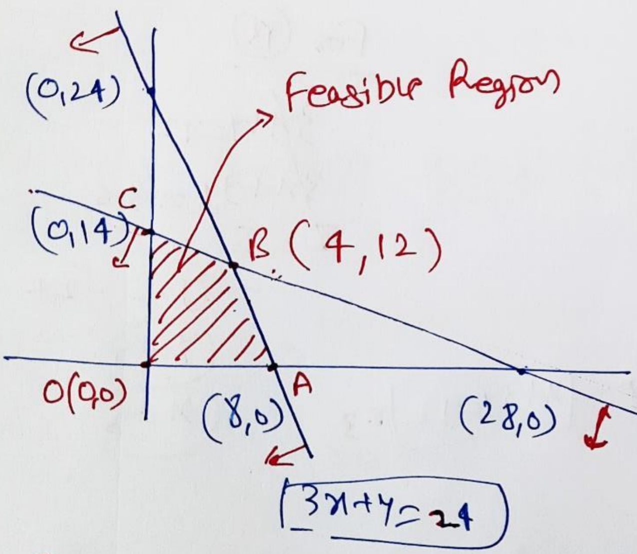
$x \geq 0$
$y \geq 0$

Inequalities.

Machine $1.5x + 3y \leq 42 \longrightarrow 1.5x+3y=42 \quad (0,14), (28,0)$

Craftman $3x + y \leq 24 \longrightarrow 3x+y=24 \quad (0,24), (8,0)$

Objective Fn: Capacity $(z) = x+y$ ~~Maxi~~ Maximise



For B

$3x+y=24$ $\times 2$...

$$\begin{aligned} 3x+y &= 24 \\ -\;\cancel{3x}+6y &= 84 \\ \hline -5y &= -60 \\ y &= 12 \\ x &= 4 \end{aligned}$$

$1.5x+3y=42$ $\times 2$

Table.

Corner Points	$Z = (x+y)$
O (0,0)	0
A (8,0)	8
B (4,12)	16
C (0,14)	14

Full Capacity

$16 = M =$ maximum.

Rackets $= 4 = x$
Bats $= 12 = y$

Maximum Profit $= 20x+10y$
$= 20 \times 4 + 10 \times 12 = 200$ ₹

Exercise 12.2 Class 12

Q.4

	Quantity	Machine 'A' time	Machine (B) time	(₹) Profit
Nuts	$x \geq 0$	1 hour	3 hours	17.50
Bolts	$y \geq 0$	3	1	7

max. 12 hrs   max. 12 hrs

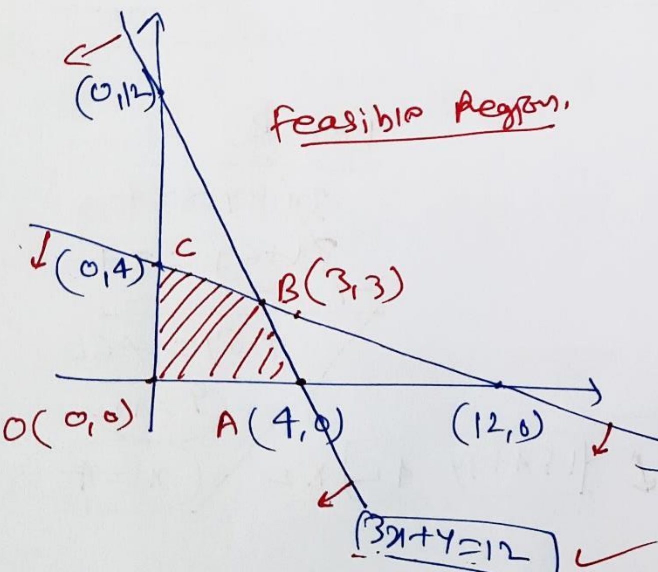
Inequality

(A) $x + 3y \leq 12 \rightarrow x+3y=12$ (0,4), (12,0)

(B) $3x + y \leq 12 \rightarrow 3x+y=12$ (0,12), (4,0)

~~Objection~~

Objective function, Profit $(z) = 17.5x + 7y$ (maximise)



For (B)

$\cancel{3x}+y=12$ ×3

$\cancel{3x}+9y=36$

$-8y=-24$

$y=3$

$x=3$

Table

Corner Points	Profit $z=17.5x+7y$
O (0,0)	0
A (4,0)	70
B (3,3)	52.5+21 = 73.5 ← maximum profit
C (0,4)	28

Profit ⟷ maximum

x = no. of packages of nuts = 3

y = ———————— Bolts = 3

Exercise 12.2

Q.5

	Quantity	Profit (₹)	Automatic (time)	Hand operated (time)
Screw (A)	x	7	4 min.	6 min.
Screw (B)	y	10	6 min.	3 min.
			↓	↓
			4 hrs (240 min.)	4 hrs. (240 min.) max.

$x, y \geq 0$

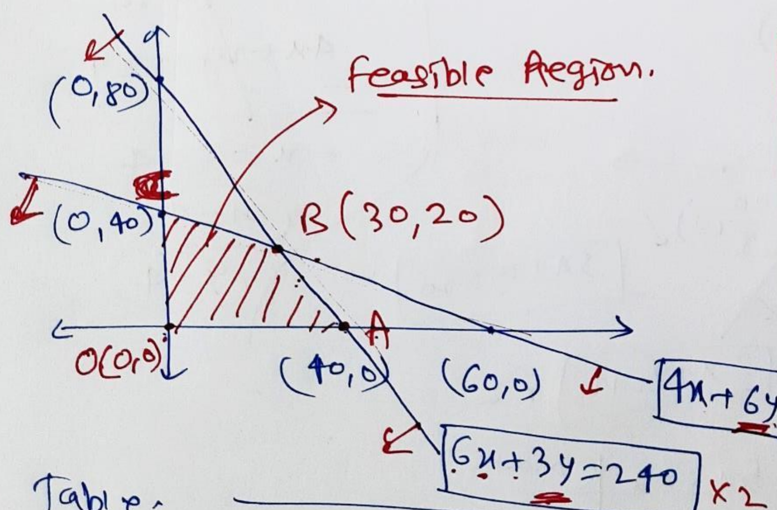
~~Data~~

Inequalities.

Automatic $\quad 4x+6y \leq 240 \longrightarrow 4x+6y=240 \quad (0,40) \quad (60,0)$

Hand operated $\quad 6x+3y \leq 240 \longrightarrow 6x+3y=240 \quad (0,80) \quad (40,0)$

Objective Function Profit $= (z) = 7x+10y$ (maximise)



For (B)

$$4x+6y=240$$
$$12x+6y=480 \quad (\times 2)$$
$$\underline{- \quad - \quad -}$$
$$+8x = +240$$
$$x = 30$$
$$y = 20$$

210
200

Table.

Corner Points	$z = (7x+10y)$ ← Profit
O (0,0)	0
A (40,0)	280
B (30,20)	410 = maximum profit.
C (0,40)	400

$x=30$, $y=20$

Screw (A) → 30
Screw (B) → 20

Exercise 12.2 Class 12

Q.6

	Quantity	Profit (₹)	Grinding/Cutting Machine (time)	Sprayer (time)
Lamps	x	5	2	3
shades	y	3	1	2
		max.?	max. 12 hrs	max. 20 hrs

Inequalities.

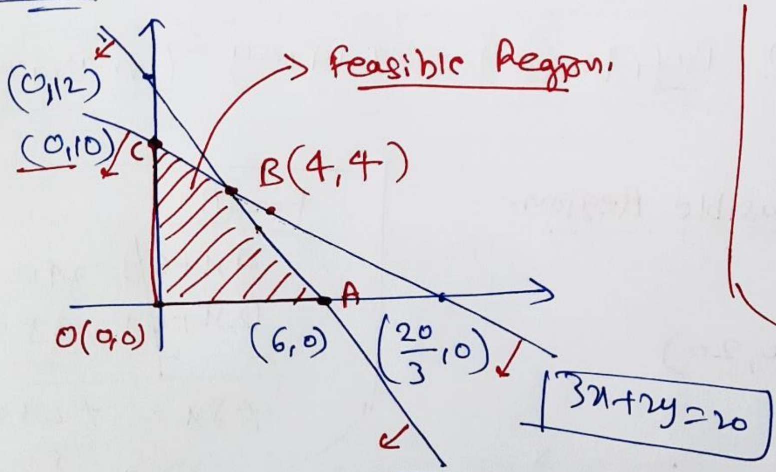
G/C $\rightarrow$ $2x+y \le 12 \rightarrow 2x+y=12$ $(0,12),(6,0)$

Sprayer $\rightarrow$ $3x+2y \le 20 \rightarrow 3x+2y=20$ $(0,10),\left(\frac{20}{3},0\right)$

$x \ge 0,\ y \ge 0 \rightarrow$ I-quadrant.

Objective Function. Profit $(Z) = 5x+3y$ (maximise)

Diagram.



For B

$3x+2y=20$

$4x+2y=24$ (×2)

$-x=-4$

$x=4$

$y=4$

Table

Corner Points	$Z=5x+3y$ (max.)
O (0,0)	0
A (6,0)	30
B (4,4)	32 $\leftarrow$ maximum profit
C (0,10)	30

$x=4$, $y=4$

maximum profit $\rightarrow$ lamps = 4, shades = 4

Exercise 12.2 | Class 12

Q.7

	Quantity	Profit (₹)	Cutting Time	Assembling Time
Type (A)	x	5	5 min.	10 min.
Type (B)	y	6	8 min.	8 min.
max. available			3 hrs 20 min $(3\times60+20)$ min. = 200 min.	4 hrs = 240 min.

Inequalities.

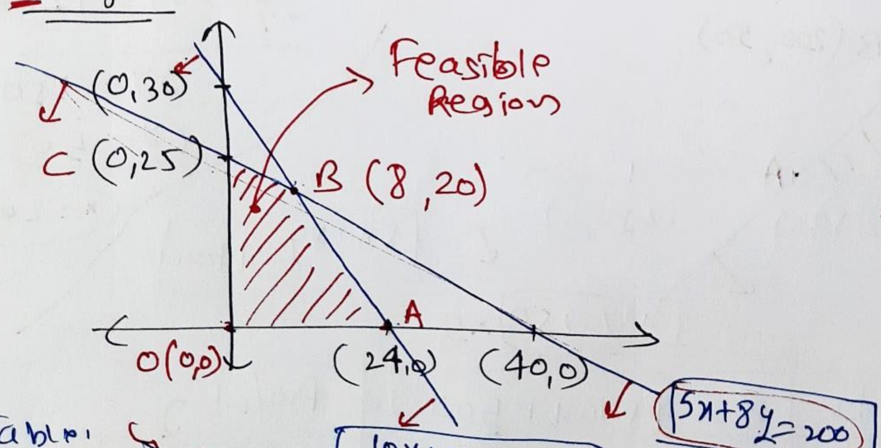
Cutting Time → $5x+8y \le 200 \rightarrow 5x+8y=200$ $(0,25)$ $(40,0)$

Assembling Time → $10x+8y \le 240 \rightarrow 10x+8y=240$ $(0,30)$ $(24,0)$

$x, y \geq 0 \rightarrow$ I quad.

Objective Function. Profit $(z) = 5x+6y$ (maximise)

Diagram.



For (B)

$$10x+\cancel{8y}=240$$
$$5x+\cancel{8y}=200$$
$$- \quad - \quad -$$
$$5x=40$$
$$x=8$$
$$y=20$$

Table.

Corner Points	$z=5x+6y$ (Profit) → max.
O (0,0)	0
A (24,0)	120
B (8,20)	160 ← max. profit.
C (0,25)	150

$x=8 \leftarrow$ type (A)

$y=20 \leftarrow$ type (B)

Exercise 12.2

Q. 8

	units	Profit	Cost
Desktop	x	4500	25000
Portable	y	5000	40000

max. ₹ 70,00,000

Inequalities.

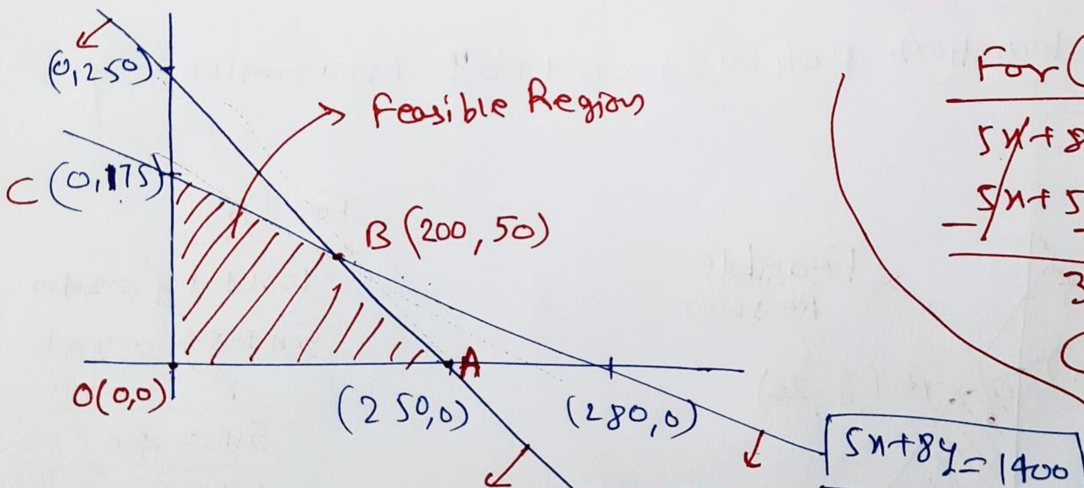
Demand → $x+y \le 250 \longrightarrow x+y=250$ $(0,250), (250,0)$

Investment → $25000x+40000y \le 7000000$

$\Rightarrow \boxed{5x+8y \le 1400} \to 5x+8y=1400$
$(0,175)$
$(280,0)$

$\boxed{x \ge 0,\ y \ge 0}$

Objective function $\boxed{\text{Profit}(z) = 4500x+5000y} \to$ maximise



For (B)

$$5x+8y=1400$$
$$\times 5 \qquad -\ 5x+5y=1250$$
$$3y=150$$
$$y=50$$
$$x=200$$

Table.

Corner Points	$z = 4500x+5000y$ ← Profit ? max.
O (0,0)	0
A (250,0)	11,25,000
B (200, 50)	11,50000 → maximum Profit
C (0,175)	8,75,000

$x = 200$ = Desktop

$y = 50$ = Portable

Exercise 12.2 | Class 12

Q.9

	Units	Cost (₹/unit)	Vitamin A	Minerals
F_1	x	4	3	4
F_2	y	6	6	3
			at least 80	100 units at least

Inequalities.

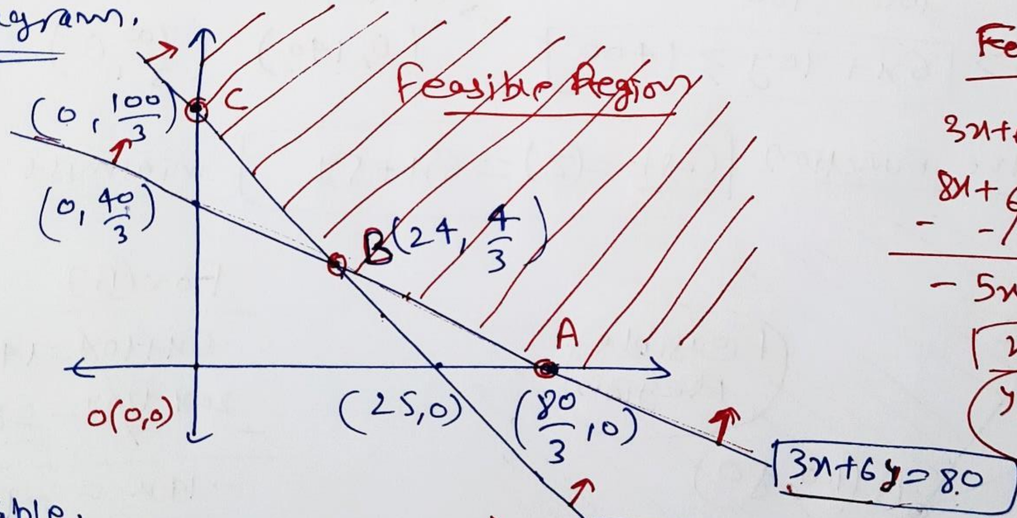
vit A → $3x+6y \geq 80 \rightarrow 3x+6y=80 \quad (0,\frac{40}{3}),(\frac{80}{3},0)$

mineral → $4x+3y \geq 100 \rightarrow 4x+3y=100 \quad (0,\frac{100}{3}),(25,0)$

$x \geq 0, y \geq 0$

Objective Function $\boxed{\text{Cost } (z) = 4x+6y}$ minimise

Diagram.



For (B)

$3x+6y=80$

$8x+6y=200$ ($4x+3y=100$ × 2)

$-5x=-120$

$\boxed{x=24}$

$y=\frac{4}{3}$

Table.

Corner Points	$z=4x+6y$
A $(\frac{80}{3},0)$	$\frac{320}{3}=106.66...$
B $(24,\frac{4}{3})$	104 = minimum Cost
C $(0,\frac{100}{3})$	200

Cost (minimise)

Exercise 12.2 Class 12

Q.10

	Quantity (Kg)	Cost (₹/Kg)	N_2	H_3PO_4
F_1	x	6	$10\% = x \times \frac{10}{100}$	$6\% = x \times \frac{6}{100}$
F_2	y	5	$5\% = y \times \frac{5}{100}$	$10\% = y \times \frac{10}{100}$

$x \geq 0, y \geq 0$

atleast 14 Kg. 14 Kg

Inequalities,

$N_2 \to x \times \frac{10}{100} + y \times \frac{5}{100} \geq 14$

$\Rightarrow \boxed{10x + 5y \geq 1400} \to 10x + 5y = 1400$

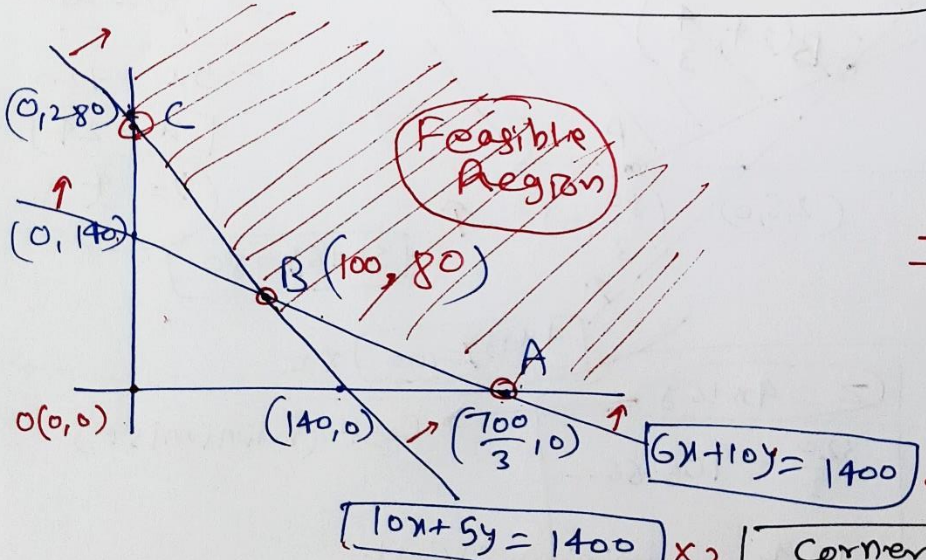
$(0, 280), (140, 0)$

$H_3PO_4 \to \frac{6x}{100} + \frac{10y}{100} \geq 14$

$\Rightarrow \boxed{6x + 10y \geq 1400} \to 6x + 10y = 1400$

$(0, 140)\ \left(\frac{700}{3}, 0\right)$

Objective Function $\boxed{\text{Cost} = (Z) = 6x + 5y}$ minimise



For (B)

$6x + 10y = 1400$

$20x + 10y = 2800$

$-14x = -1400$

$\boxed{x = 100}$

$\boxed{y = 80}$

Cost (↓)

Corner Points	$Z = 6x + 5y$
A $\left(\frac{700}{3}, 0\right)$	1400
B (100, 80)	1000
C (0, 280)	1400

minimum Cost = 1000

$x = 100$ Kg ← (F_1)

$y = 80$ kg ← (F_2)

Exercise 12.2, Class 12

Q.11

$2x + y \le 10$
$x + 3y \le 15$
$x, y \ge 0$

$Z = px + qy$ $\quad p, q > 0$

$\Rightarrow$ maximum of Z occurs at Both (3, 4) and (0, 5) is —

(A) $p = q$ (B) $p = 2q$
(C) $p = 3q$ (D) $q = 3p$

Corner Points
(0,0) (5,0) (3,4), (0,5)

I O D F C T

Table:

Corner Points	$Z = px + qy$
(0,0)	0
(5,0)	5p
(3,4)	3p + 4q ← maximum
(0,5)	5q ← maximum

$3p + 4q = 5q$
$\Rightarrow 3p = 5q - 4q$
$\Rightarrow 3p = q$

Miscellaneous Exercise on Chapter 12

Q.1

	No. of Packets	Ca	Fe	Cholesterol	Vitamin A
Food P	x	12	4	6	6
Food Q	y	3	20	4	3
		min. 240	min. 460	max. 300	maximise?

$x, y \geq 0$ → I quad.

Inequalities

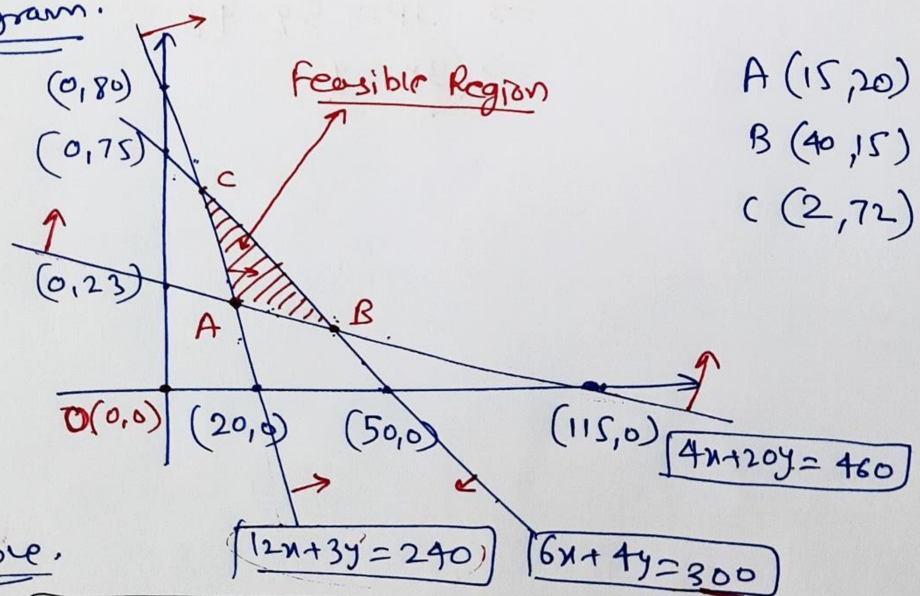
Ca: $12x + 3y \geq 240 \rightarrow 12x + 3y = 240$ (0, 80) (20, 0)

Fe: $4x + 20y \geq 460 \rightarrow 4x + 20y = 460$ (0, 23) (115, 0)

Chol.: $6x + 4y \leq 300 \rightarrow 6x + 4y = 300$ (0, 75) (50, 0)

$$\boxed{\text{Objective Function, Vit. A} = Z = 6x + 3y \ \text{(maximise)}}$$

Diagram.



A (15, 20)

B (40, 15)

C (2, 72)

Table.

Corner Points	$Z = 6x + 3y$ max. (V.A)
A (15, 20)	90 + 60 = 150
B (40, 15)	240 + 45 = 285 = max. Vit. A
C (2, 72)	12 + 216 = 228

x = 40 = Food P

y = 15 = Food Q

Q.2

	No of Bags	Cost (₹)	N.E. (A)	(B)	(C)
Brand (P)	x	250	3	2.5	2
Brand (Q)	y	200	1.5	11.25	3
		minimise	18	45	24

$x, y \geq 0$

min. requirement → Inequality

Inequalities.

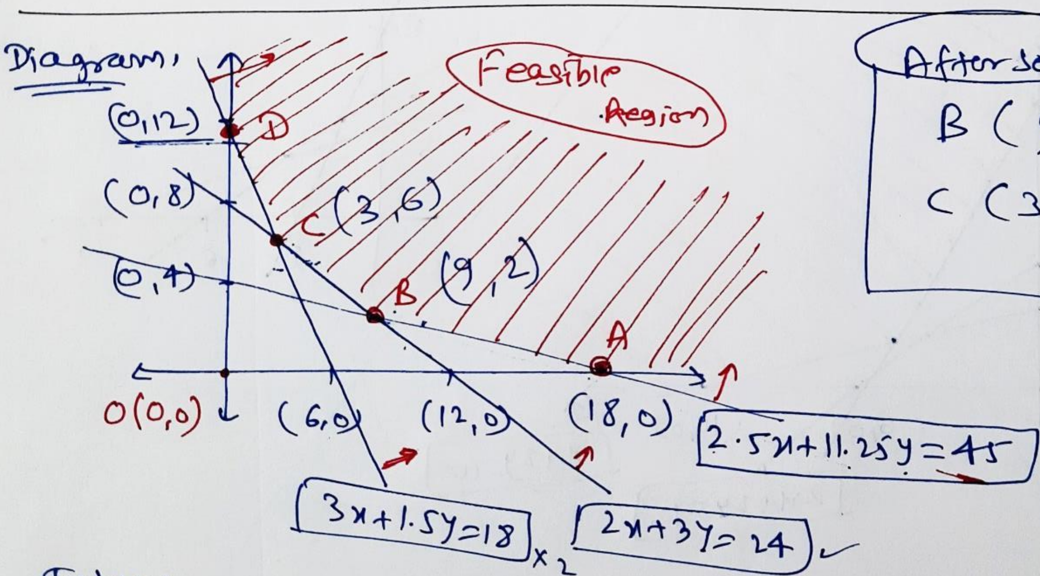
(A) → $3x + 1.5y \geq 18$ → $3x + 1.5y = 18$ (0, 12) (6, 0)

(B) → $2.5x + 11.25y \geq 45$ → $2.5x + 11.25y = 45$ (0, 4) (18, 0)

(C) → $2x + 3y \geq 24$ → $2x + 3y = 24$ (0, 8) (12, 0)

Objective Function: $\boxed{\text{Cost } (z) = 250x + 200y}$ minimise

Diagram:



After solving

B (9, 2)

C (3, 6)

Table:

Corner Points	$z = 250x + 200y$ → cost min.
A (18, 0)	4500
B (9, 2)	2250 + 400 = 2650
C (3, 6)	750 + 1200 = 1950 = minimum cost
D (0, 12)	0 + 2400 = 2400

$x = 3$ ← (P)

$y = 6$ ← (Q)

Q.3

Food	Quantity (Kg)	Cost (₹)	vit A	vit B	vit C
X	x	16	1	2	3
Y	y	20	2	2	1
$x, y \geq 0$		minimise	10	12	8

minimum requirement

Inequalities,

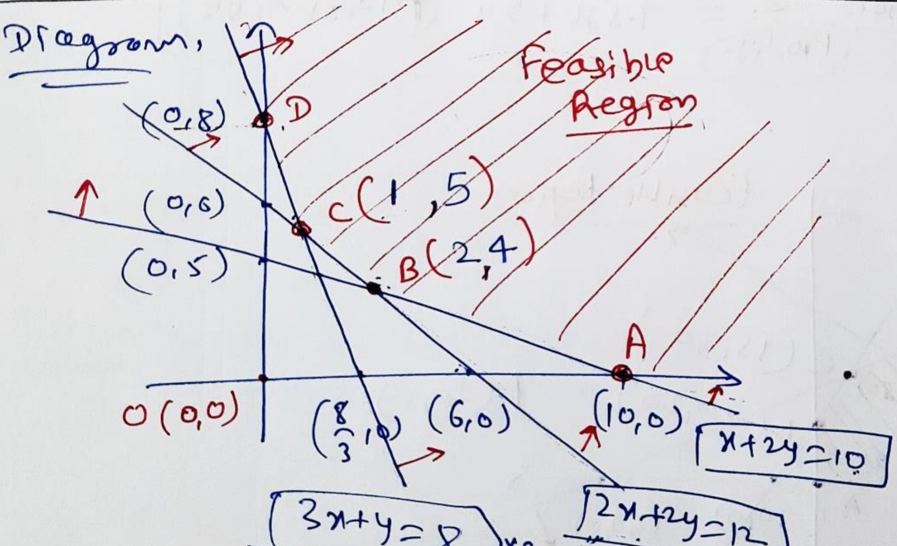
Vit A → $x+2y \geq 10$ → $x+2y=10$ → $(0,5), (10,0)$

Vit B → $2x+2y \geq 12$ → $2x+2y=12$ → $(0,6), (6,0)$

Vit C → $3x+y \geq 8$ → $3x+y=8$ → $(0,8), (\frac{8}{3},0)$

Objective Function, $\boxed{\text{Cost } (Z) = 16x+20y}$ minimise

Diagram,



B (2,4)

C (1,5)

Table

Corner Points	$Z = 16x+20y$ (Cost) minimum
A (10,0)	$160+0=160$
B (2,4)	$32+80=112$ ⇒ minimum Cost
C (1,5)	$16+100=116$
D (0,8)	$0+160=160$

$X = 2$ Kg

$Y = 4$ Kg

112

miscellaneous Ex. chapter 12 Class 12

Q.4

Types of Toys	No. of toys	Profit (₹)	minutes machines I	II	III
A	x	7.5	12	18	6
B	y	5	6	0	9
		maximum	6 hrs	6 hrs	6 hrs
		Maximum availability	360 min.	360 min	360 min.

$x \geq 0$, $y \geq 0$ → I-quad

Inequality,

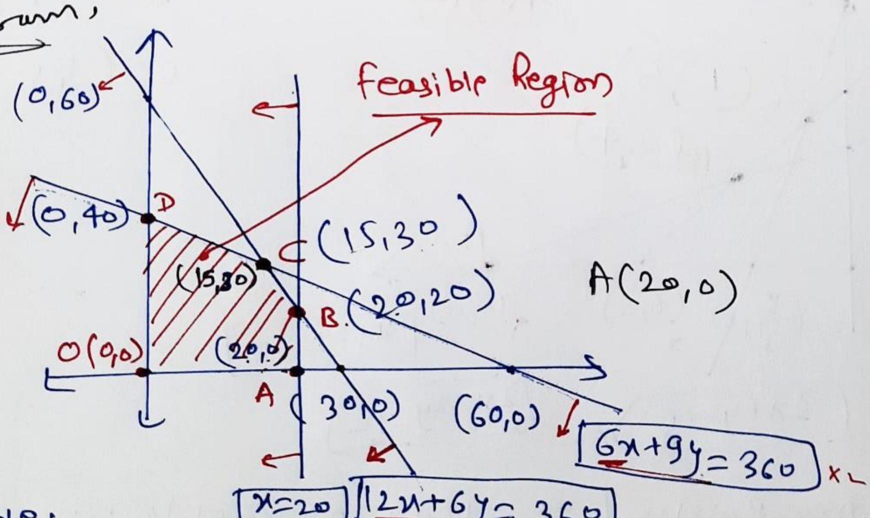
machine I: $12x + 6y \leq 360 \rightarrow 12x + 6y = 360$ (0, 60), (30, 0)

machine II: $18x + 0.y \leq 360 \rightarrow 18x = 360 \rightarrow \boxed{x = 20}$

machine III: $6x + 9y \leq 360 \rightarrow 6x + 9y = 360$ (0, 40), (60, 0)

Objective function $Z = 7.5x + 5y$ (Maximise)
Profit

Diagram,



Table,

Corner Points	$Z = 7.5x + 5y$ Profit (maximum)
O (0,0)	0
A (20,0)	150
B (20,20)	250
C (15,30)	262.5 → maximum Profit
D (0,40)	200

at C(15, 30)
x → Type A, y → B

Miscellaneous Exercise on Chapter (12)

Q.5

	No. of Tickets	Profit (₹ / ticket)
Executive Class	x	1000
Economy Class	y	600

No. of passengers that can be carried by Aeroplane = max. (200)

$$\boxed{x+y \le 200}$$

Atleast 20 seats for Executive class $\Rightarrow$ $\boxed{x \ge 20}$

Passengers in economy class = atleast 4 times of passengers in Executive class.

$$\boxed{y \ge 4x}$$ $x \ge 0$, $y \ge 0$

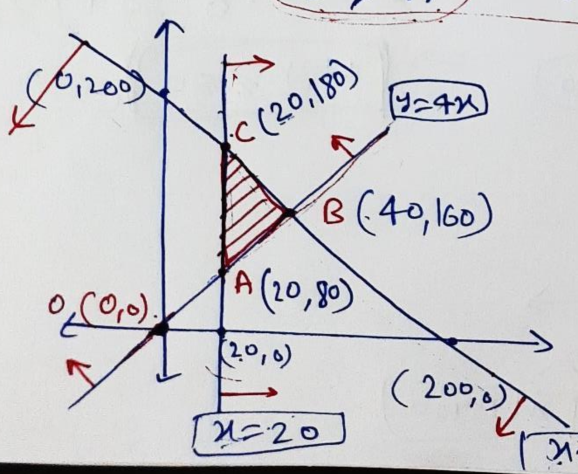
Objective Function: $\boxed{\text{Profit } (Z) = 1000x + 600y}$ maximise

Diagram

$x+y \le 200 \longrightarrow x+y = 200$ $(0,200)$, $(200,0)$

$x \ge 20 \longrightarrow \boxed{x = 20}$

$y \ge 4x \longrightarrow \boxed{y = 4x}$ $(0,0)$ Slope $m = 4$ Positive



Table

Corner Points	$Z = 1000x + 600y$
A (20, 80)	68 000
B (40, 160)	136000
C (20, 180)	128 000

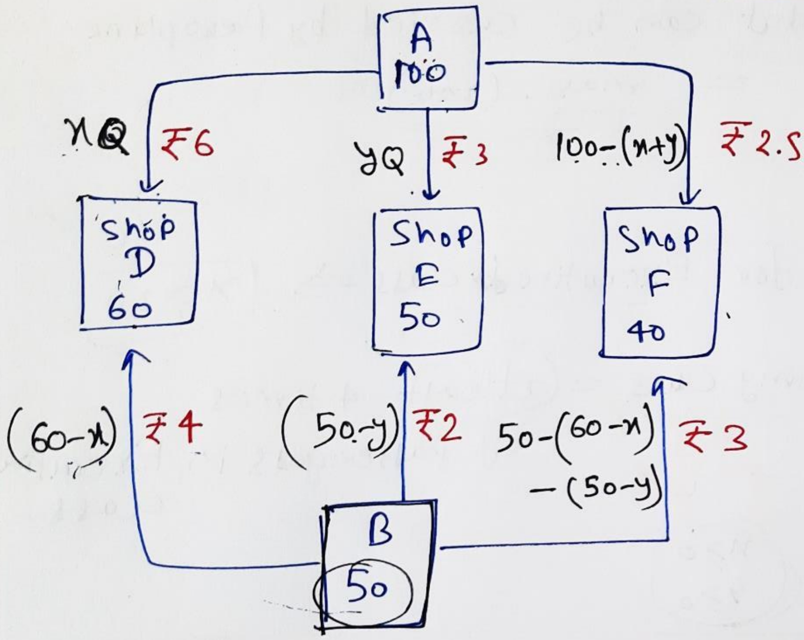
$x = 40$, $y = 160$ $\longrightarrow$ max. Profit

Q.6 Capacity of Godown A = 100 Q | Capacity of Godown B = 50 Q

Demand of Shop D = 60 Q
Demand of Shop E = 50 Q
Demand of Shop F = 40 Q

Transportation Cost / Quintal (₹)

From / To	A	B
D	6	4
E	3	2
F	2.5	3



Inequalities:

- $x \geq 0$
- $y \geq 0$

Supply ≥ 0

- $100 - (x+y) \geq 0$

$\Rightarrow 100 \geq (x+y)$

$\Rightarrow \boxed{x + y \leq 100}$

- $60 - x \geq 0$

$\Rightarrow \boxed{x \leq 60}$

- $50 - y \geq 0$

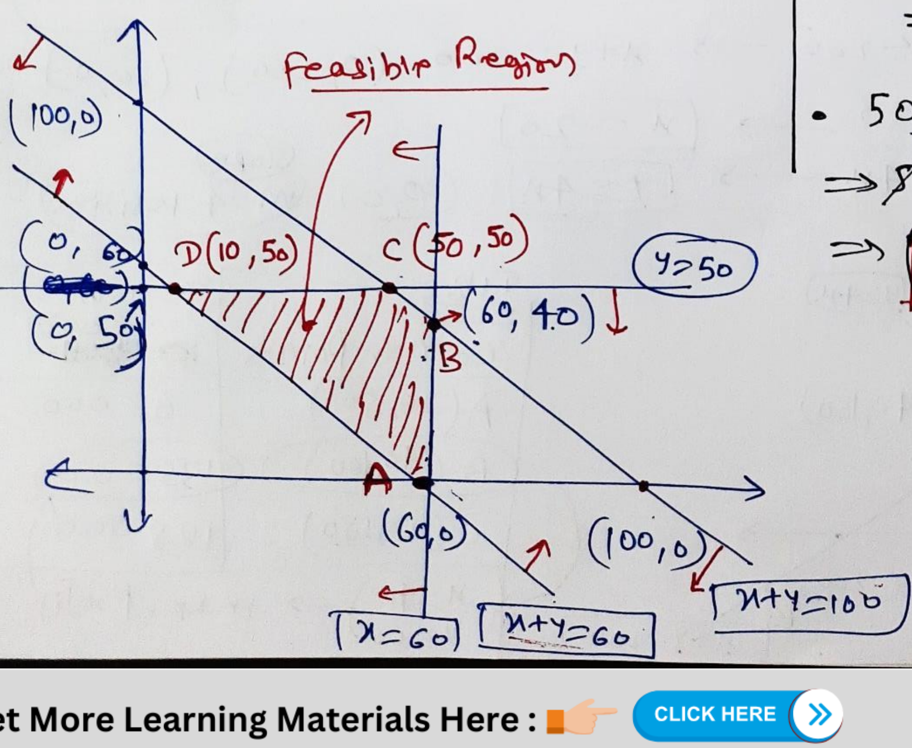
$\Rightarrow \boxed{y \leq 50}$

- $50 - (60 - x) - (50 - y) \geq 0$

$\Rightarrow \cancel{50} - 60 + x - \cancel{50} + y \geq 0$

$\Rightarrow \boxed{x + y \geq 60}$

Diagram



Objective Function (Z) = [Transportation Cost]

(minimise)

$$Z = 6x + 3y + [100 - (x+y)](2.5) + (60-x)\cdot 4 + (50-y)\cdot 2 + (x+y-60)\times 3$$

$$Z = 6x + 3y + \boxed{250} - 2.5x - 2.5y + \boxed{240} - 4x + \boxed{100} - 2y + 3x + 3y - \boxed{180}$$

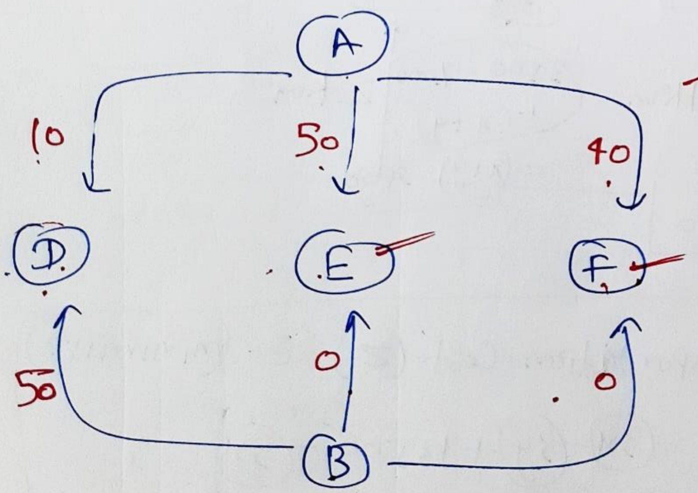
$$Z = 2.5x + 1.5y + 410$$

Table

Corner Points	$Z = 2.5x + 1.5y + 410$
A (60, 0)	$150 + 410 = 560$
B (60, 40)	$150 + 60 + 410 = 620$
C (50, 50)	$125 + 75 + 410 = 610$
D (10, 50)	$25 + 75 + 410 = 510$

x y

minimum Cost



After Putting $x = 10$ & $y = 50$

Q.7 | Capacity of Depot 'A' = 7000L | Capacity of Depot 'B' = 4000L |

Requirement of Petrol Pump D = 4500 L

Requirement of Petrol Pump E = 3000 L

Requirement of Petrol Pump F = 3500 L

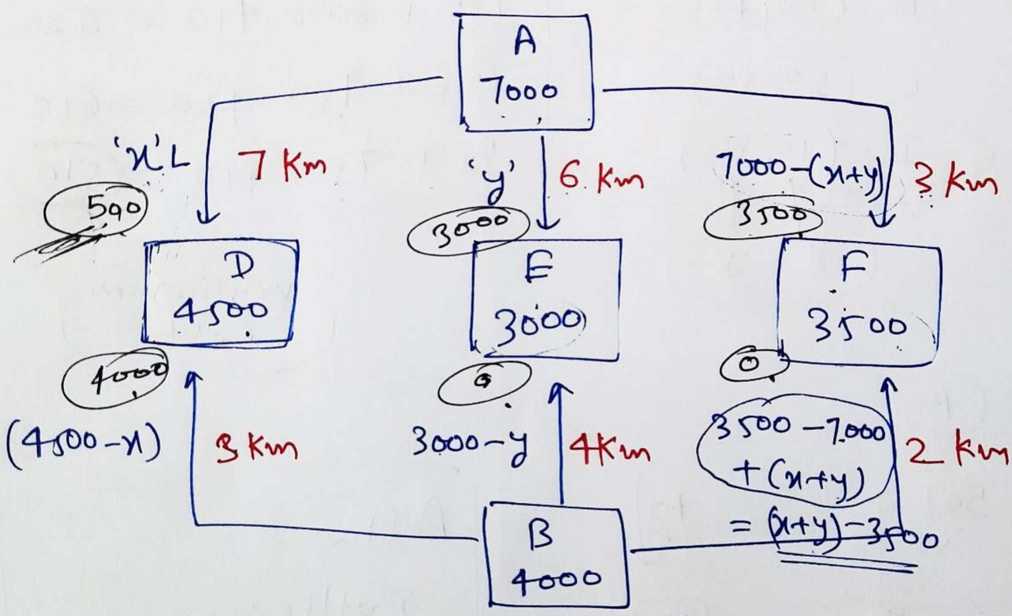
Distance (Km)

From/To	A	B
D	7	3
E	6	4
F	3	2

Transportation cost of 10 litres of oil = ₹1/km

Minimum Transportation Cost = ?

⇒ Transportation Cost of 1 litre of oil = ₹ $\frac{1}{10}$/km



Objective function Transportation Cost (Z) ← (minimise)

$$Z = \left(7x + 6y + 21000 - 3x - 3y + 13500 - 3x + 12000 - 4y + 2x + 2y - 7000\right) \times \frac{1}{10}$$

$$= (3x + y + 39500)\frac{1}{10} = 0.3x + 0.1y + 3950$$

Inequalities.

$\boxed{\text{Supply} \geq 0}$

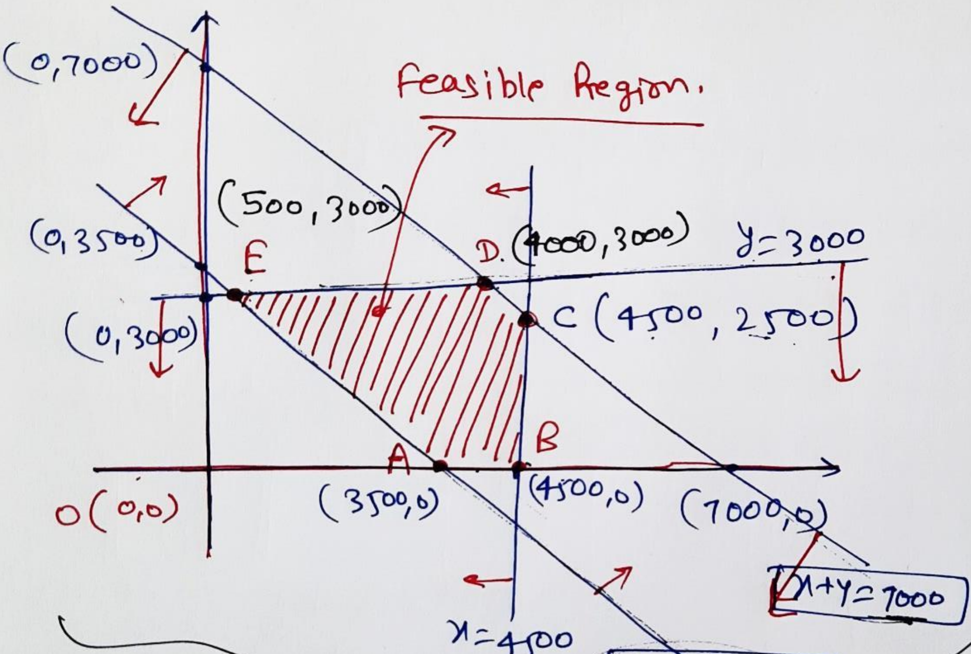
$x \geq 0$, $y \geq 0$ } → I-quadrant

→ $7000-(x+y) \geq 0$ → $x+y \leq 7000$ → $x+y = 7000$ (0, 7000), (7000, 0)

$4500 - x \geq 0$ → $x \leq 4500$ → $x = 4500$

$3000 - y \geq 0$ → $y \leq 3000$ → $y = 3000$

→ $x+y-3500 \geq 0$ → $x+y \geq 3500$ → $x+y = 3500$ (0, 3500), (3500, 0)



Table

Corner Points.	Cost (minimise) $Z = 0.3x + 0.1y + 3950$
A (3500, 0)	1050 + 0 + 3950
B (4500, 0)	1350 + 0 + 3950
C (4500, 2500)	1350 + 250 + 3950
D (4000, 3000)	1200 + 300 + 3950
E (500, 3000)	150 + 300 + 3950

↑ x ↑ y

= minimum cost = 4400 ₹

Miscellaneous Exercise on Chapter 12

Q.8 → minimum amount of N_2 added

Q.9 → maximum amount of N_2 added.

	No. of Bags	N_2	H_3PO_4	Potash	Chlorine
Brand (P)	x	3	1	3	1.5
Brand (Q)	y	3.5	2	1.5	2
$x, y \geq 0$		?	min (240 kg)	min. (270 kg)	max (310) kg.

Inequalities:

H_3PO_4 → $x + 2y \geq 240 \longrightarrow x + 2y = 240$ $(0, 120), (240, 0)$

Potash → $3x + 1.5y \geq 270 \longrightarrow 3x + 1.5y = 270$ $(0, 180), (90, 0)$

Cl_2 → $1.5x + 2y \leq 310 \longrightarrow 1.5x + 2y = 310$ $(0, 155), \left(\frac{620}{3}, 0\right)$

Objective function Nitrogen $(Z) = 3x + 3.5y$ → max. → Q.9; → min → Q.8

N_2

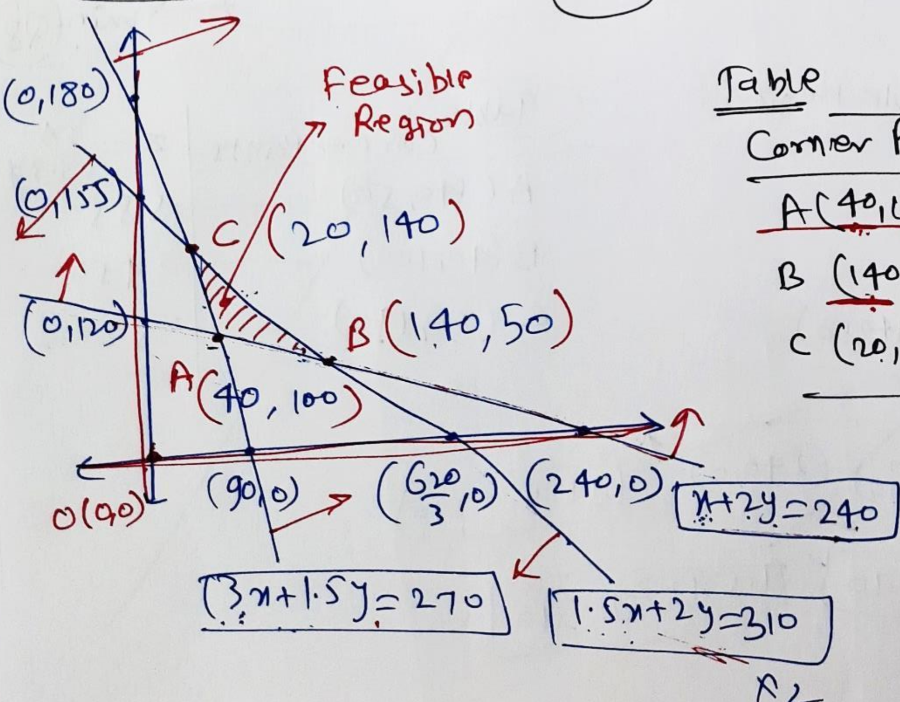


Table N_2

Corner Points	$Z = 3x + 3.5y$
A (40, 100)	470 = min.
B (140, 50)	595 max.
C (20, 140)	550

Q.9

Miscellaneous Exercise on Chapter (12) Class (12)

Q.10

	No. of Dolls	Profit (₹/Doll)
Doll 'A' →	x	12
Doll 'B' →	y	16

$x, y \geq 0$

Objective function Profit (z)

$Z = 12x + 16y$ maximise

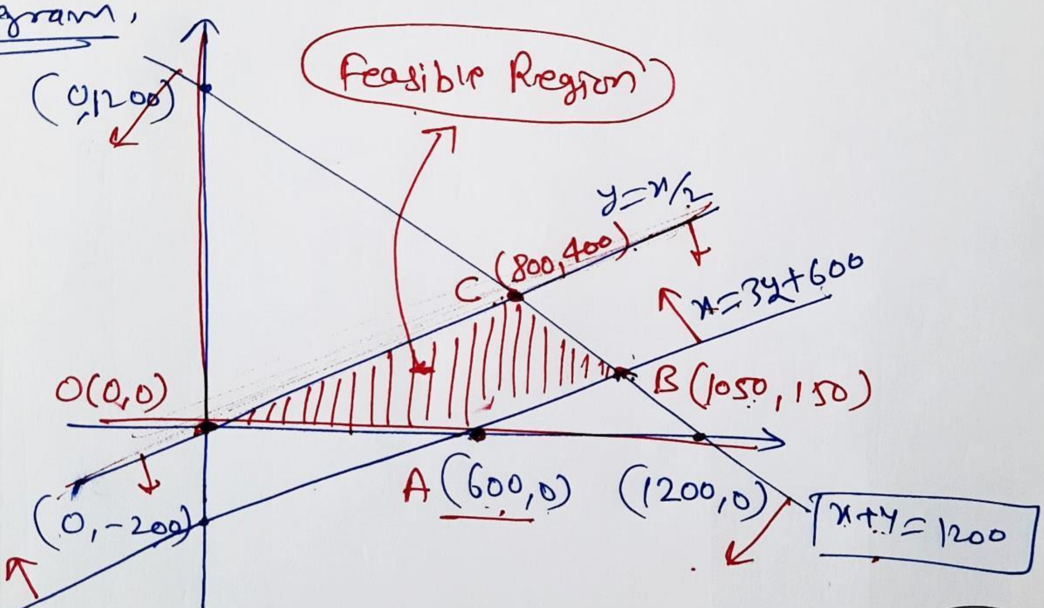
Inequalities

$x + y \leq 1200 \longrightarrow x + y = 1200 \quad (0, 1200), \ (1200, 0)$

$y \leq \frac{x}{2} \longrightarrow y = \frac{x}{2} \quad (0,0)$ Slope = (+) $(1050, 525)$

$x \leq 3y + 600 \longrightarrow x = 3y + 600 \quad (0, -200), \ (600, 0)$

Diagram,



Table

Corner Points	$Z = 12x + 16y$
O (0, 0)	0
A (600, 0)	7200
B (1050, 150)	15000
C (800, 400)	16000 = maximum profit

Profit maximise

C (800, 400) → x → Doll (A), y → Doll (B)